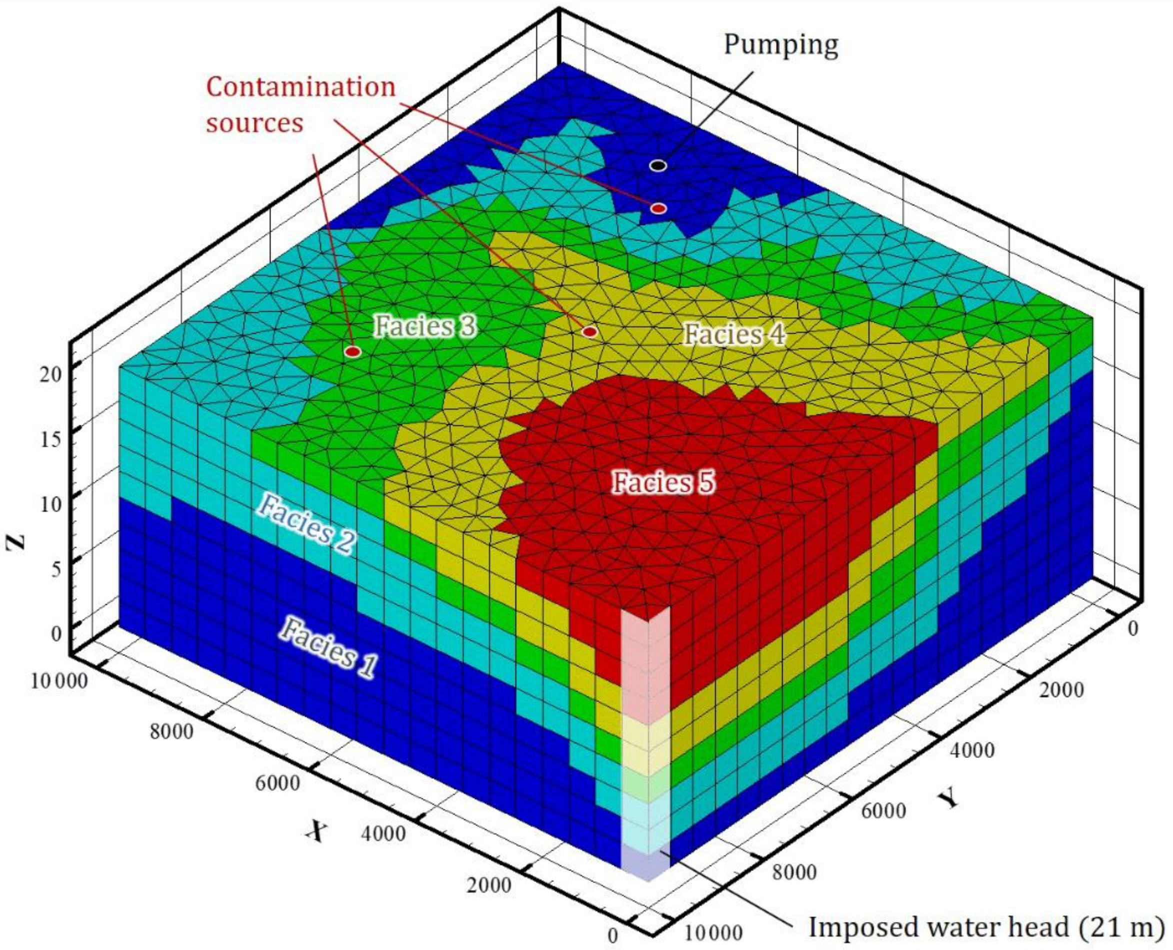
Water Influx Models

- Pot aquifer
- Schilthuis' steady-state
- Hurst's modified steady-state
- The Van Everdingen-Hurst unsteady-state
- The Carter-Tracy unsteady-state
- Fetkovich's method



Introduction

Limited information: During exploration, information about potential aquifers is often scarce.

Clues from performance: Early signs of water influx come from reservoir performance data.

Indicators:

- **Slower pressure decline:** Compared to withdrawal, if pressure decline is slow and decreasing, it suggests influx.
- **Pressure change analysis:** Tracking barrels withdrawn per pressure drop can support this observation.
- **Reservoir boundaries:** If boundaries are unknown, influx might be from an unmapped area.
- **Gas-oil ratio:** If reservoir pressure is below oil saturation pressure, a slow increase in gas-oil ratio indicates influx.
- **Early water production:** Water from edge wells suggests encroachment, but fractures, streaks, or limited aquifer influence need to be ruled out.
- **Material balance:** If original oil-in-place estimates increase with pressure surveys (assuming no influx), it hints at influx.

The Pot Aquifer Model

Simplest model: Assumes constant pressure at the aquifer boundary and no pressure decline within the reservoir.

Suitable for early estimates: Useful for quick assessments when detailed data is limited.

Limitations: Doesn't capture pressure decline or dynamic behavior of the aquifer.

The Pot Aquifer Model

The simplest model that can be used to estimate the water influx into a gas or oil reservoir is based on the basic definition of compressibility:

$$\Delta V = c\ V\ \Delta p$$

→

$$W_e = (c_w + c_f) W_i (p_i - p)$$

→

$$W_i = \left[\frac{\pi (r_a^2 - r_e^2) h\ \phi}{5.615}\right]$$

↓

$$r_e = \sqrt{\frac{360\ V_P}{\pi h \phi \theta}}$$

←

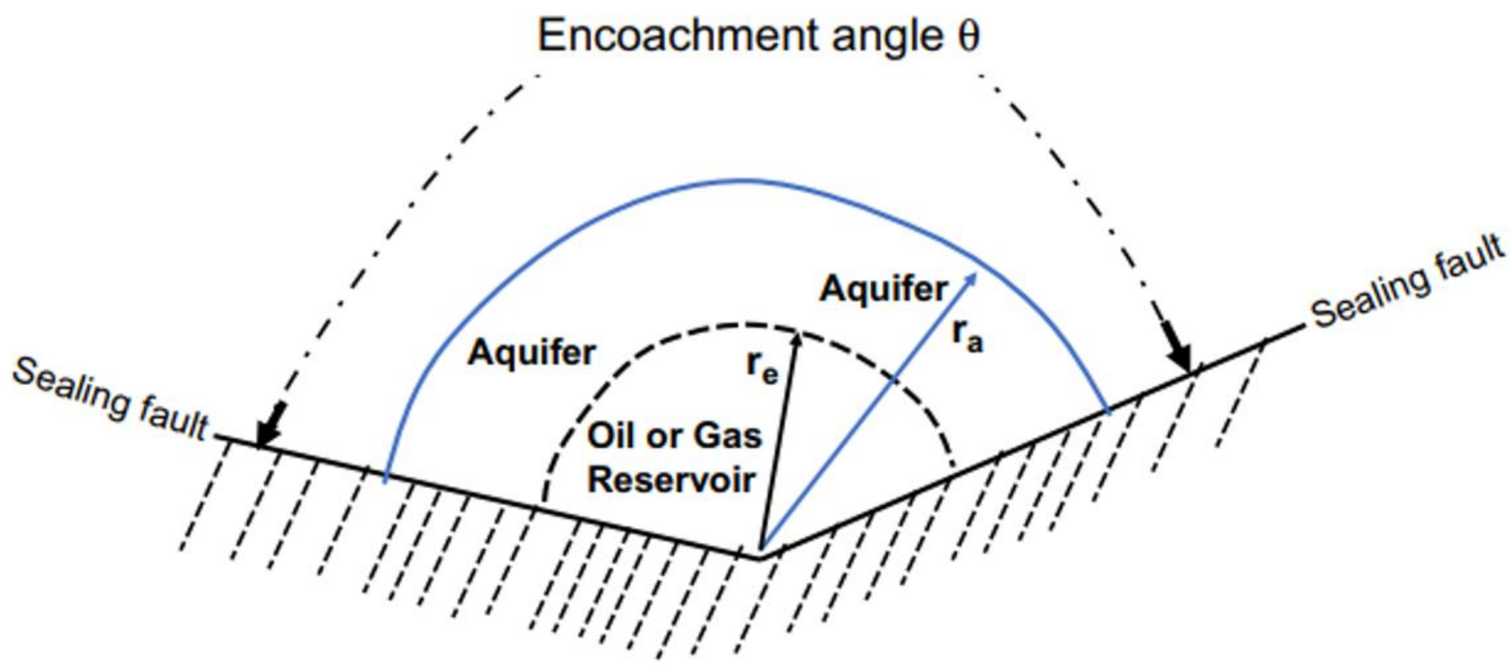
$$W_e = (c_w + c_f)\ W_i\ f\ (p_i - p)$$

←

$$f = \frac{(\text{encoachment angle})^\circ}{360^\circ} = \frac{\theta}{360^\circ}$$

The Pot Aquifer Model

The above model is only applicable to a small aquifer, i.e., pot aquifer, whose dimensions are of the same order of magnitude as the reservoir itself.



The Pot Aquifer Model

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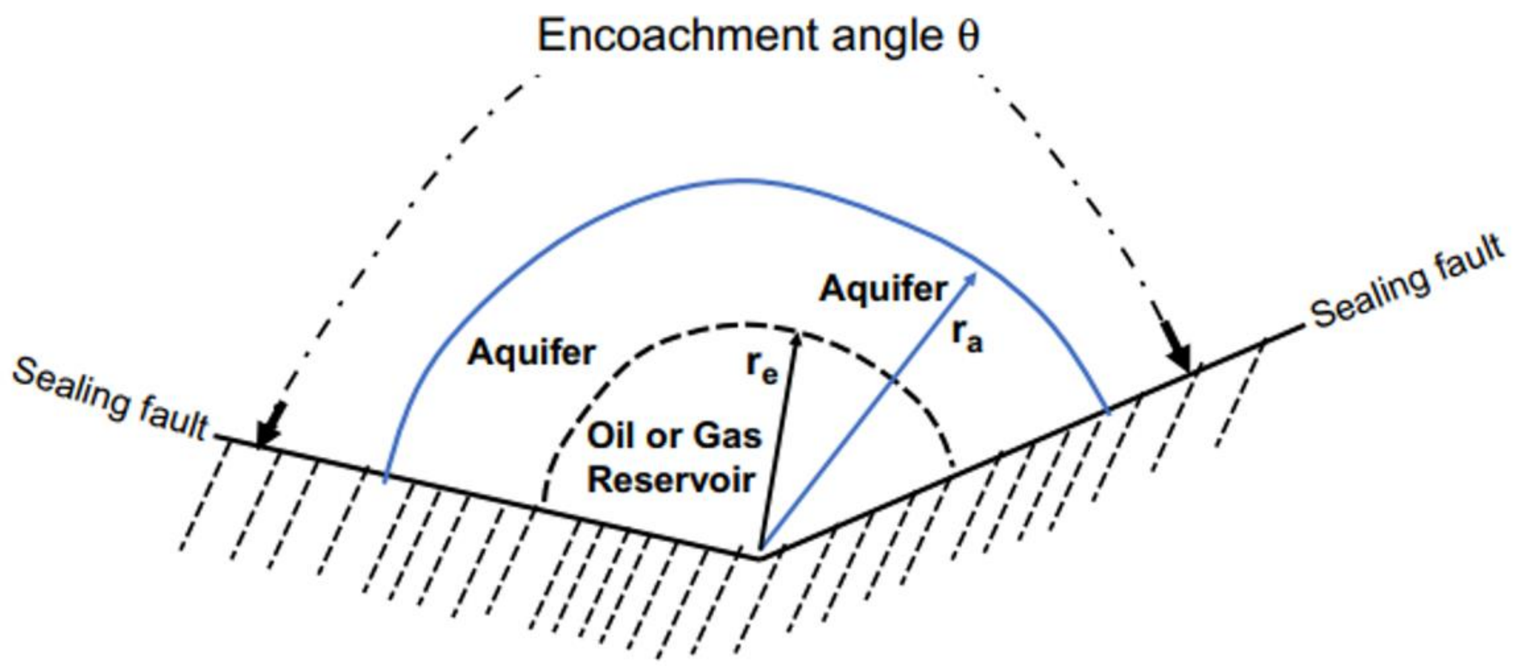


Schilthuis' Stead -State Model

The flow behavior: could be described by Darcy's equation

More realistic: Provides a better approximation than the Pot model.

Limitations: Still assumes steady-state and doesn't consider transient behavior.

Schilthuis' Stead -State Model

$$\frac{dW_e}{dt} = e_w = \left[\frac{0.00708\ kh}{\mu_w\ \ln\left(\frac{r_a}{r_e}\right)}\right](p_i - p)$$

The above relationship can be more conveniently expressed as:

$$\frac{dW_e}{dt} = e_w = C(p_i - p)$$

where

e_w = rate of water influx, bbl/day
k = permeability of the aquifer, md
h = thickness of the aquifer, ft
r_a = radius of the aquifer, ft
r_e = radius of the reservoir
t = time, days

This water influx constant C may be calculated from the reservoir historical production data over a number of selected time intervals, provided that the rate of water influx ew has been determined independently from a different expression

Schilthuis' Stead -State Model

Note that the pressure drops contributing to influx are the cumulative pressure drops from the initial pressure.

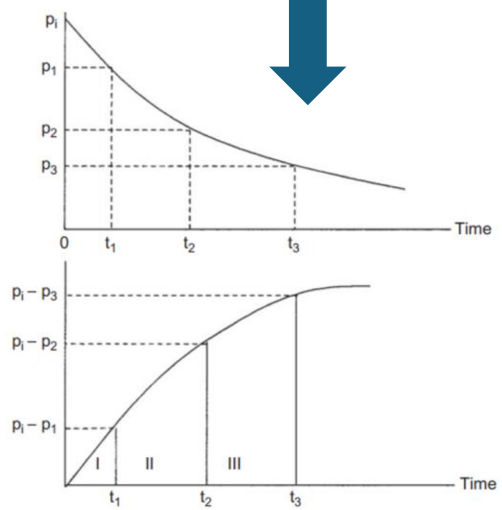
$$\frac{dW_e}{dt} = e_w = C(p_i - p)$$

In terms of the cumulative water influx We,

$$W_e = C \int_o^t (p_i - p)\ dt$$

This area at time t can be determined numerically by using the trapezoidal rule (or any other numerical integration method), as:

$$\int_o^t (p_i - p) = dt = \text{area}_I + \text{area}_{II} + \text{area}_{III} + \text{etc.} = \left(\frac{p_i - p_1}{2}\right)(t_1 - 0)$$
$$+ \frac{(p_i - p_1) + (p_i - p_2)}{2}(t_2 - t_1) + \frac{(p_i - p_2) + (p_i - p_3)}{2}(t_3 - t_2)$$
$$+ \text{etc.}$$



Hurst's Modified Stead -State Model

Refines Schilthuis' model: Introduces a time factor to account for the transient period before steady-state is reached.

Improves accuracy: Offers a more realistic representation of early production stages.

Limitations: Assumes steady-state eventually and may not be suitable for long-term predictions.

Hurst's Modified Stead -State Model

One of the problems associated with the Schilthuis' steady-state model is that as the water is drained from the aquifer, the aquifer drainage radius ra will increase as the time increases.

Hurst (1943) proposed that the "apparent" aquifer radius ra would increase with time and, therefore the **dimensionless radius ra/re** may be replaced with a time dependent function.

The Hurst modified steady-state equation contains two unknown constants, i.e., a and C, that must be determined from the reservoir-aquifer pressure and water influx historical data.

$$\frac{dW_e}{dt} = e_w = \left[\frac{0.00708\ kh}{\mu_w \ln\left(\frac{r_a}{r_e}\right)}\right](p_i - p)$$

$$r_a / r_e = at$$

$$e_w = \frac{dW_e}{dt} = \frac{0.00708\ kh(p_i - p)}{\mu_w \ln(at)}$$

$$e_w = \frac{dW_e}{dt} = \frac{C(p_i - p)}{\ln(at)}$$

$$W_e = C\sum_{o}^{t}\left[\frac{\Delta p}{\ln(at)}\right]\Delta t$$

Hurst's Modified Stead -State Model

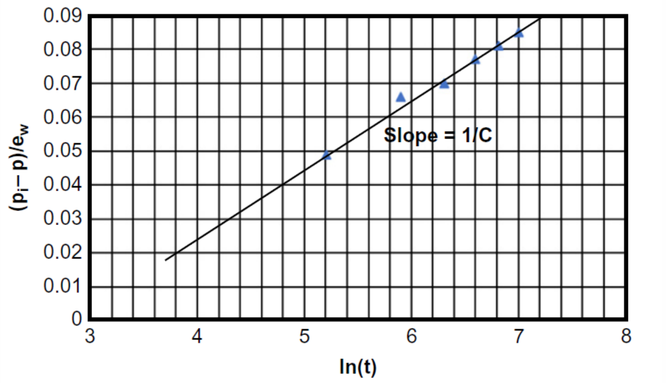
The Hurst modified steady-state equation contains two unknown constants, i.e., a and C, that must be determined from the reservoir-aquifer pressure and water influx historical data. The procedure of determining the constants a and C is as the following:

$$e_w = \frac{dW_e}{dt} = \frac{C(p_i - p)}{\ln(at)}$$

expressing this Equation as a linear relationship

$$\frac{p_i - p}{e_w} = \left(\frac{1}{C}\right) \ln(a) + \left(\frac{1}{C}\right) \ln(t)$$

plot of (pi – p)/ew versus ln(t) will be a straight line with a **slope of 1/C** and intercept of **(1/C)ln(a)**



Van Everdingen-Hurst Unsteady-State Model

Similar to Van Everdingen-Hurst: Analyzes unsteady-state behavior but uses different mathematical formulations.

Offers advantages: Can be computationally simpler for certain situations.

Limitations: Less widely used and may not be as accurate as Van Everdingen-Hurst in all cases.

Van Everdingen-Hurst Unsteady-State Model

The mathematical formulations that describe the flow of crude oil system into a wellbore are identical in form to those equations that describe the flow of water from an aquifer into a **cylindrical reservoir**

→

The dimensionless form of the diffusivity equation is basically the general mathematical equation that is designed to model the transient flow behavior in reservoirs or aquifers. In a dimensionless form, **the diffusivity equation** takes the form:

→

Van Everdingen and Hurst (1949) proposed solutions to the dimensionless diffusivity equation for the following two reservoir-aquifer boundary conditions:

1. Constant terminal rate
2. Constant terminal pressure

→

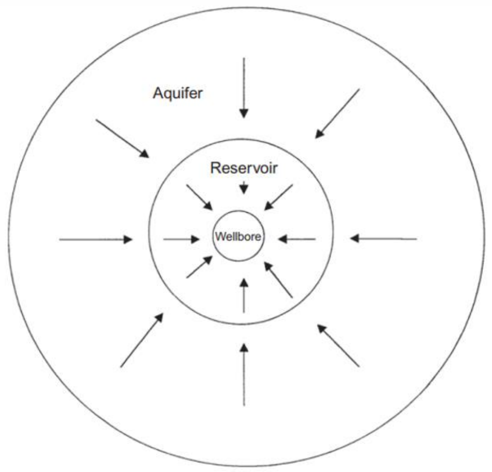
Van Everdingen and Hurst solved the diffusivity equation for the aquifer reservoir system by applying the Laplace transformation to the equation.

→

The solution can be used to determine the water influx in the following systems:

1- Edge-water-drive system (radial system)

2- Bottom-water-drive system



$$\frac{\partial^2 P_D}{\partial r_D^2} + \frac{1}{r_D}\frac{\partial P_D}{\partial r_D} = \frac{\partial P_D}{\partial t_D}$$

Van Everdingen-Hurst Unsteady-State Model

Edge-Water Drive

$$W_e = B \Sigma \Delta P X Q t$$

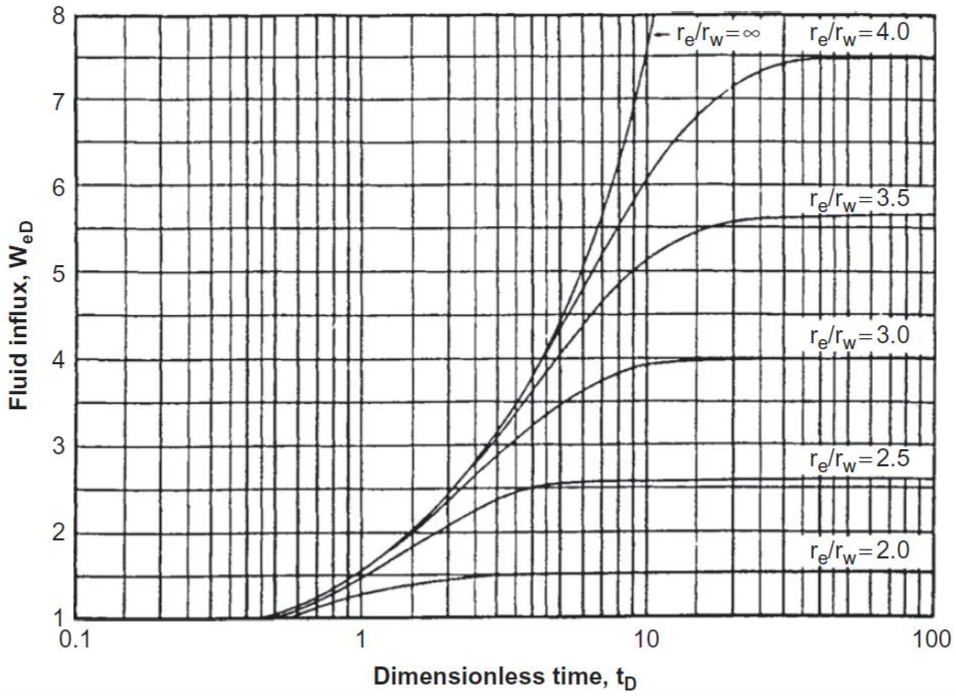
$$B = 1.119\,\phi\, c_t\, r_e^2 h\, f$$

$$f = \frac{\theta}{360}$$

$$t_D = 6.328 \times 10^{-3} \frac{kt}{\phi \mu_w\, c_t\, r_e^2}$$

$$r_D = \frac{r_a}{r_e}$$

Q_t *dimensionless flow rate* $= f(t_D)$ *[table]*



Van Everdingen-Hurst Unsteady-State Model

Edge-Water Drive – Infinite Aquifer

$$Q_t \quad dimensionless\ flow\ rate = f(t_D) \quad [table]$$

TABLE 10-1 Dimensionless Water Influx W_{eD} for Infinite Aquifer

Dimensionless time t_D	Fluid influx W_{eD}	Dimensionless time t_D	Fluid influx W_{eD}	Dimensionless time t_D	Fluid influx W_{eD}	Dimensionless time t_D	Fluid influx W_{eD}	Dimensionless time t_D	Fluid influx W_{eD}	Dimensionless time t_D	Fluid influx W_{eD}
0.00	0.000	79	35.697	455	150.249	1190	340.843	3250	816.090	35.000	6780.247
0.01	0.112	80	36.058	460	151.640	1200	343.308	3300	827.088	40.000	7650.096
0.05	0.278	81	36.418	465	153.029	1210	345.770	3350	838.067	50.000	9363.099
0.10	0.404	82	36.777	470	154.416	1220	348.230	3400	849.028	60.000	11,047.299
0.15	0.520	83	37.136	475	155.801	1225	349.460	3450	859.974	70.000	12,708.358
0.20	0.606	84	37.494	480	157.184	1230	350.688	3500	870.903	75.000	13,531.457
0.25	0.689	85	37.851	485	158.565	1240	353.144	3550	881.816	80.000	14,350.121
0.30	0.758	86	38.207	490	159.945	1250	355.597	3600	892.712	90.000	15,975.389
0.40	0.898	87	38.563	495	161.322	1260	358.048	3650	903.594	100.000	17,586.284
0.50	1.020	88	38.919	500	162.698	1270	360.496	3700	914.459	125.000	21,560.732
0.60	1.140	89	39.272	510	165.444	1275	361.720	3750	925.309	$1.5(10)^5$	$2.538(10)^4$
0.70	1.251	90	39.626	520	168.183	1280	362.942	3800	936.144	2.0″	3.308″
0.80	1.359	91	39.979	525	169.549	1290	365.386	3850	946.966	2.5″	4.066″
0.90	1.469	92	40.331	530	170.914	1300	367.828	3900	957.773	3.0″	4.817″
1	1.569	93	40.684	540	173.639	1310	370.267	3950	968.566	4.0″	6.267″
2	2.447	94	41.034	550	176.357	1320	372.704	4000	979.344	5.0″	7.699″
3	3.202	95	41.385	560	179.069	1325	373.922	4050	990.108	6.0″	9.113″
4	3.893	96	41.735	570	181.774	1330	375.139	4100	1000.858	7.0″	$1.051(10)^5$

Van Everdingen-Hurst Unsteady-State Model

Edge-Water Drive – Finite Aquifer

$$Q_t \quad dimensionless\ flow\ rate = f(t_D) \quad [table]$$

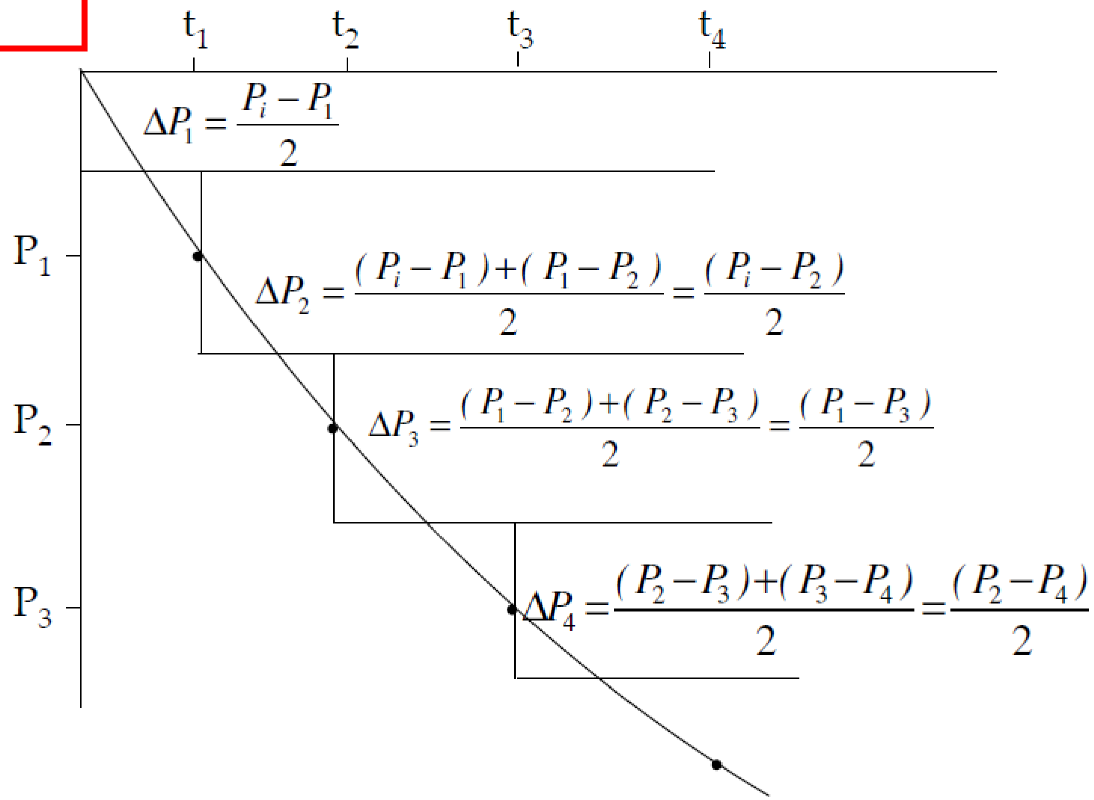
TABLE 10-2 Dimensionless Water Influx W_{eD} for Several Values of r_e/r_R, i.e. r_a/r_e—cont'd

$r_e/r_R = 5.0$		$r_e/r_R = 6.0$		$r_e/r_R = 7.0$		$r_e/r_R = 8.0$		$r_e/r_R = 9.0$		$r_e/r_R = 10.0$	
Dimensionless time t_D	Fluid influx W_{eD}	Dimensionless time t_D	Fluid influx W_{eD}	Dimensionless time t_D	Fluid influx W_{eD}	Dimensionless time t_D	Fluid influx W_{eD}	Dimensionless time t_D	Fluid influx W_{eD}	Dimensionless time t_D	Fluid influx W_{eD}
3.0	3.195	6.0	5.148	9.00	6.861	9	6.861	10	7.417	15	9.965
3.5	3.542	6.5	5.440	9.50	7.127	10	7.398	15	9.945	20	12.32
4.0	3.875	7.0	5.724	10	7.389	11	7.920	20	12.26	22	13.22
4.5	4.193	7.5	6.002	11	7.902	12	8.431	22	13.13	24	14.95
5.0	4.499	8.0	6.273	12	8.397	13	8.930	24	13.98	26	14.95
5.5	4.792	8.5	6.537	13	8.876	14	9.418	26	14.79	28	15.78
6.0	5.074	9.0	6.795	14	9.341	15	9.895	26	15.59	30	16.59
6.5	5.345	9.5	7.047	15	9.791	16	10.361	30	16.35	32	17.38
7.0	5.605	10.0	7.293	16	10.23	17	10.82	32	17.10	34	18.16
7.5	5.854	10.5	7.533	17	10.65	18	11.26	34	17.82	36	18.91
8.0	6.094	11	7.767	18	11.06	19	11.70	36	18.52	38	19.65
8.5	6.325	12	8.220	19	11.46	20	12.13	38	19.19	40	20.37
9.0	6.547	13	8.651	20	11.85	22	12.95	40	19.85	42	21.07
9.5	6.760	14	9.063	22	12.58	24	13.74	42	20.48	44	21.76
10	6.965	15	9.456	24	13.27	26	14.50	44	21.09	46	22.42
11	7.350	16	9.829	26	13.92	28	15.23	46	21.69	48	23.07

Van Everdingen-Hurst Unsteady-State Model

Edge-Water Drive – Calculation of ΔP

$$\Delta P_n = \frac{P_{n-2} - P_n}{2}$$

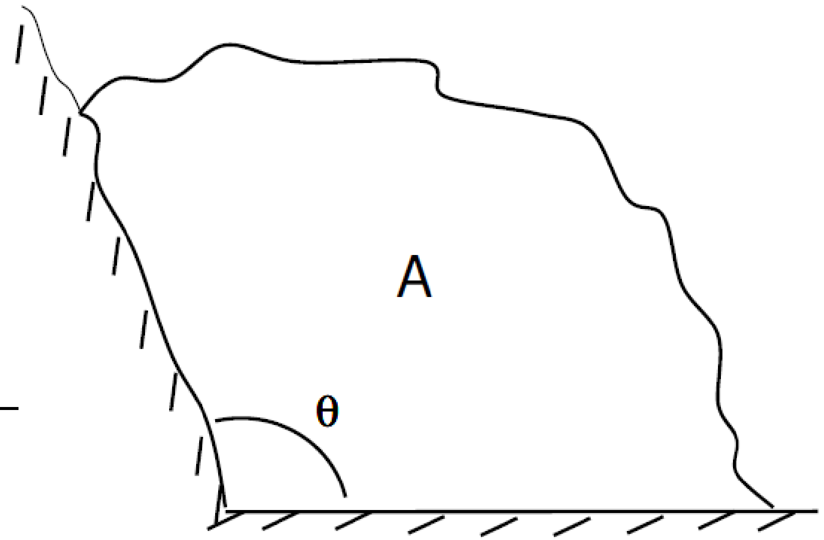


T_{month}	0	6	12	18	24
P_{psi}	2500	2490	2472	2444	2408

The time intervals must all be equal in order to preserve the accuracy of these modifications.

Van Everdingen-Hurst Unsteady-State Model

Edge-Water Drive – Calculation of (r_e)



$$A = \pi r^2 \times \frac{\theta}{360}$$

$$\therefore r_w = \sqrt{\frac{360 A}{\pi \theta}}$$

Van Everdingen-Hurst Unsteady-State Model

Bottom-Water Drive

The van Everdingen-Hurst solution to the radial diffusivity equation is considered the most rigorous aquifer influx model to date. The proposed solution technique, however, is not adequate to describe the vertical water encroachment in bottom-water-drive system

Coats 1962 presented a mathematical model that takes into account the vertical flow effects from bottom-water aquifers. He correctly noted that in many cases reservoirs are situated on top of an aquifer with a continuous horizontal interface between the reservoir fluid and the aquifer water and with a significant aquifer thickness.

Coats modified the diffusivity equation to account for the vertical flow by including an additional term in the equation, to give:

$$\frac{\partial^2 p}{\partial r^2} + \frac{1}{r}\frac{\partial p}{\partial r} + F_k \frac{\partial^2 p}{\partial z^2} = \frac{\mu \phi c}{k}\frac{\partial p}{\partial t}$$

$$F_k = k_v / k_h$$

where

k_v = vertical permeability
k_h = horizontal permeability

$$z_D = \frac{h}{r_e \sqrt{F_k}}$$

where

z_D = dimensionless vertical distance
h = aquifer thickness, ft

Van Everdingen-Hurst Unsteady-State Model

Bottom-Water Drive

$$W_e = B \Sigma \Delta P X Q t$$

$$B = 1.119\,\phi\, c_t\, r_e^2\, h$$

$$t_D = 6.328 \times 10^{-3} \frac{kt}{\phi \mu_w\, c_t\, r_e^2}$$

$$r_D = \frac{r_a}{r_e}$$

$$\frac{\partial^2 p}{\partial r^2} + \frac{1}{r}\frac{\partial p}{\partial r} + F_k \frac{\partial^2 p}{\partial z^2} = \frac{\mu \phi c}{k}\frac{\partial p}{\partial t}$$

$$F_k = k_v / k_h$$

$$z_D = \frac{h}{r_e \sqrt{F_k}}$$

where

k_v = vertical permeability
k_h = horizontal permeability

where

z_D = dimensionless vertical distance
h = aquifer thickness, ft

Van Everdingen-Hurst Unsteady-State Model

Bottom-Water Drive – Infinite Aquifer

$$Q_t = f(t_D, r_D \text{ and } z_D)$$

TABLE 10-3 Dimensionless Water Influx, W_{eD}, for Infinite Aquifer *(Permission to publish by the SPE)*

	z'_D						
t_D	*0.05*	*0.1*	*0.3*	*0.5*	*0.7*	*0.9*	*1.0*
0.1	0.700	0.677	0.508	0.349	0.251	0.195	0.176
0.2	0.793	0.786	0.696	0.547	0.416	0.328	0.295
0.3	0.936	0.926	0.834	0.692	0.548	0.440	0.396
0.4	1.051	1.041	0.952	0.812	0.662	0.540	0.486
0.5	1.158	1.155	1.059	0.918	0.764	0.631	0.569
0.6	1.270	1.268	1.167	1.021	0.862	0.721	0.651
0.7	1.384	1.380	1.270	1.116	0.953	0.806	0.729
0.8	1.503	1.499	1.373	1.205	1.039	0.886	0.803
0.9	1.621	1.612	1.477	1.286	1.117	0.959	0.872
1	1.743	1.726	1.581	1.347	1.181	1.020	0.932
2	2.402	2.393	2.288	2.034	1.827	1.622	1.509
3	3.031	3.018	2.895	2.650	2.408	2.164	2.026
4	3.629	3.615	3.477	3.223	2.949	2.669	2.510
5	4.217	4.201	4.048	3.766	3.462	3.150	2.971
6	4.784	4.766	4.601	4.288	3.956	3.614	3.416
7	5.323	5.303	5.128	4.792	4.434	4.063	3.847
8	5.829	5.808	5.625	5.283	4.900	4.501	4.268

Van Everdingen-Hurst Unsteady-State Model

Bottom-Water Drive – Finite Aquifer

$$Q_t = f(t_D, r_D \text{ and } z_D)$$

TABLE 10-4 Dimensionless Water Influx, W_{eD}, for $r'_D = 4$ (*Permission to publish by the SPE*)—cont'd

	z'_D						
t_D	*0.05*	*0.1*	*0.3*	*0.5*	*0.7*	*0.9*	*1.0*
60	7.968	7.968	7.965	7.954	7.931	7.898	7.864
70	7.976	7.976	7.976	7.968	7.965	7.942	7.920
80	7.982	7.982	7.987	7.976	7.976	7.965	7.954
90	7.987	7.987	7.987	7.984	7.983	7.976	7.965
100	7.987	7.987	7.987	7.987	7.987	7.983	7.976
120	7.987	7.987	7.987	7.987	7.987	7.987	7.987

The Carter-Tracy Water Influx Model

Similar to Van Everdingen-Hurst: Analyzes unsteady-state behavior but uses different mathematical formulations.

Offers advantages: Can be computationally simpler for certain situations.

Limitations: Less widely used and may not be as accurate as Van Everdingen-Hurst in all cases.

The Carter-Tracy Water Influx Model

The primary difference between the Carter-Tracy technique and the van Everdingen-Hurst technique is that the Carter-Tracy technique assumes constant water influx rates over each finite time interval.

Using the Carter-Tracy technique, the cumulative water influx at any time, tn, can be calculated directly from the previous value obtained at tn – 1;

Carter-Tracy method considerably overestimates the water influx. Accuracy of the Carter-Tracy method can be increased substantially by restricting the time step used in performing the water influx calculations to small time intervals

$$(W_e)_n = (W_e)_{n-1} + \left[(t_D)_n - (t_D)_{n-1}\right] \left[\frac{B\Delta p_n - (W_e)_{n-1}\,(p'_D)_n}{(p_D)_n - (t_D)_{n-1}\,(p'_D)_n}\right]$$

where

- B = the van Everdingen-Hurst water influx constant as defined by Equation 10-23
- t_D = the dimensionless time as defined by Equation 10-17
- n = refers to the *current* time step
- $n - 1$ = refers to the *previous* time step
- Δp_n = total pressure drop, $p_i - p_n$, psi
- p_D = dimensionless pressure
- p'_D = dimensionless pressure derivative

The Carter-Tracy Water Influx Model

The Dimensionless Pressure Drop (pD)

TABLE 6-2 p_D vs. t_D–Infinite-Radial System, Constant-Rate at the Inner Boundary (*After Lee, J.,* Well Testing, *SPE Textbook Series.*) (*Permission to publish by the SPE, copyright SPE, 1982*)

t_D	p_D	t_D	p_D	t_D	p_D
0	0	0.15	0.3750	60.0	2.4758
0.0005	0.0250	0.2	0.4241	70.0	2.5501
0.001	0.0352	0.3	0.5024	80.0	2.6147
0.002	0.0495	0.4	0.5645	90.0	2.6718

TABLE 6-3 p_D vs. t_D–Finite-Radial System, Constant-Rate at the Inner Boundary (*After Lee, J.,* Well Testing, *SPE Textbook Series.*) (*Permission to publish by the SPE, copyright SPE, 1982*)

$r_{eD} = 1.5$		$r_{eD} = 2.0$		$r_{eD} = 2.5$		$r_{eD} = 3.0$		$r_{eD} = 3.5$		$r_{eD} = 4.0$	
t_D	p_D	t_D	p_D	t_D	p_D	t_D	p_D	t_D	p_D	t_D	p_D
0.06	0.251	0.22	0.443	0.40	0.565	0.52	0.627	1.0	0.802	1.5	0.927
0.08	0.288	0.24	0.459	0.42	0.576	0.54	0.636	1.1	0.830	1.6	0.948

Edwardson and coauthors (1962) developed the following approximation of pD for an infinite-acting aquifer.

$$p_D = \frac{370.529\sqrt{t_D} + 137.582\, t_D + 5.69549(t_D)^{1.5}}{328.834 + 265.488\sqrt{t_D} + 45.2157\, t_D + (t_D)^{1.5}}$$

$t_D > 100$:

$$p_D = 0.5\,[\mathrm{Ln}(t_D) + 0.80907]$$

$$P'_D = 1/(2t_D)$$

Fetkovich’s Method

Pseudo-steady-state approach: Assumes constant influx rate over short time intervals but allows for changing rates over longer periods.

Simpler than Van Everdingen-Hurst: Useful for quick analysis and history matching.

Limitations: Less accurate for long-term predictions and complex reservoir geometries.

The Carter-Tracy Water Influx Model

Fetkovich (1971) developed a method of describing the approximate water influx behavior of a finite aquifer for radial and linear geometries.

The Fetkovich theory is much simpler, and, like the Carter-Tracy technique, this method does not require the use of superposition. Hence, the application is much easier, and this method is also often utilized in numerical simulation models.

Fetkovich (1971) developed a method of describing the approximate water influx behavior of a finite aquifer for radial and linear geometries.

$$e_w = \frac{dW_e}{dt} = J(\overline{p}_a - p_r)$$

where

e_w = water influx rate from aquifer, bbl/day
J = productivity index for the aquifer, bbl/day/psi
$\overline{p}_a$ = average aquifer pressure, psi
p_r = inner aquifer boundary pressure, psi

Type of Outer Aquifer Boundary	J for Radial flow, bbl/day/psi	J for Linear Flow, bbl/day/psi
Finite, no flow	$J = \frac{0.00708\ kh\ f}{m[\ln e_D - 0.75]}$	$J = \frac{0.003381\ kwh}{mL}$
Finite, constant pressure	$J = \frac{0.00708\ kh\ f}{m[\ln(r_D)]}$	$J = \frac{0.001127\ k\ wh}{mL}$
Infinite	$J = \frac{0.00708\ kh\ f}{m\ \ln(a/r_e)}$ $a = \sqrt{0.0142kt/(fmc_t)}$	$J = \frac{0.001\ k\ wh}{\sqrt[m]{0.0633kt/(fmc_t)}}$

The Carter-Tracy Water Influx Model Steps

$$W_i = \frac{\pi}{5.615}(r_a^2 - r_e^2)h\,\phi$$

$$(W_e)_{max} = c_t\, W_i\, p_i\, f$$

Calculate the productivity index J based on the boundary conditions and aquifer geometry."Table"

$$(\overline{p}_r)_1 = \frac{p_i + (p_r)_1}{2}$$

$$(\Delta W_e)_1 = \frac{(W_e)_{max}}{p_i}\left[p_i - (\overline{p}_r)_1\right]\left[1 - \exp\left(\frac{-Jp_i\,\Delta t_1}{(W_e)_{max}}\right)\right]$$

$$(\overline{p}_a)_1 = p_i\left(1 - \frac{(\Delta W_e)_1}{(W_e)_{max}}\right)$$

$$(\Delta W_e)_2 = \frac{(W_e)_{max}}{p_i}\left[(\overline{p}_a)_1 - (\overline{p}_r)_2\right]\left[1 - \exp\left(\frac{-Jp_i\Delta t_2}{(W_e)_{max}}\right)\right]$$

$$W_e = \sum_{t=1}^{n}(\Delta W_e)_i$$